UNIVERSIDAD INTERAMERICANA PARA EL DESARROLLO. UNID
CAMPUS TUXPAN, VER
ING. SOFTWARE Y SISTEMAS COMPUTACIONALES
SEMANA 3 ACTIVIDADES
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MODULO:
ALGEBRA LINEAL Y CALCULO VECTORIAL
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Combinaciones Lineal

Un vector V es combinación lineal de un conjunto si existen escalares tales que permitan escribir

$V = k_1 U_1 + k_2 U_2 + k_3 V_3 \ldots + k_n U_n$

$V = (1, -2, 5)$ $U_1 = (1, 1, 1)$ $U_2 = (1, 2, 3)$ $U_3 = (2, -1, 1)$

$$\begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + k_2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + k_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_1 \\ k_1 \end{bmatrix} + \begin{bmatrix} k_2 \\ 2k_2 \\ 3k_2 \end{bmatrix} + \begin{bmatrix} 2k_3 \\ -k_3 \\ k_3 \end{bmatrix}$$

$1 \quad k_1 + k_2 + 2k_3$ $\quad k_1 = 6 \quad k_2 = 2 \quad k_3 = 2$

$-2 = k_1 + 2k_2 - k_3$

$5 \quad k_1 + 3k_2 + k_3$

① $U_1 = (6, 0, -1)$
$U_2 = (1, 0, 0)$
$U_3 = (-6, -1, 7)$
$U_4 = (29, 9, -58)$

② $U_1 = 8, 3, 4$
$V_2 = 3, -9, -2$
$V_2 = -3, -72, -26$

④ $V_1 = (4, 1, -3)$
$V_2 = (2, 1, -2)$
$V_3 = (3, -4, 3)$
$V = (47, -32, 13)$

$U(-4, -1, -12)$ $V = (-4, -5, 6)$

$U \cdot V\ (-4)(-4) + (-1)(-5) + (-12)(6)$

$16 + 5 - 72$

-51

$\|U\| \sqrt{(-4)^2 + (-1)^2 + (-12)^2}$

$\|U\| \sqrt{16 + 1 + 144}$

$\sqrt{161}$

$\|V\| \sqrt{(-4)^2 + (-5)^2 + 6^2}$

$\|V\| \sqrt{16 + 25 + 36}$

$\|V\| \sqrt{77}$

$\frac{-51}{\|\sqrt{161}\|^2}(-4, -1, -12)$

$\frac{-51}{\|\sqrt{77}\|^2}(-4, -5, 6)$

$\frac{-51}{161} * -4, -1, -12$

$\frac{-51}{77} * -4, -5, 6$

Proyeccion V.U

$\frac{204}{161}, \frac{51}{161}, \frac{612}{161}$

Proyeccion U.V

$\frac{204}{77}, \frac{255}{77}, \frac{-306}{77}$

Sacar Coseno del angulo de este y el anterior

$\|U\| \sqrt{(-4)^2 + (-1)^2 + (-12)^2}$

$\sqrt{16 + 1 + 144}$

$\sqrt{161} = 12.68 = 13$

$\|V\| \sqrt{(-4^2 + (-5)^2 + 6^2}$

$\sqrt{16 + 25 + 36}$

$\sqrt{77} = 8.77 = 9$

$\cos\theta = \frac{-51}{9 \cdot 13} = \frac{-51}{117} = -0.43589$

$\cos^{-1}\ 64.158$

18 Sep 2024

Producto Interno (escalar, punto)

Es una multiplicacion entre 2 vectores en la misma direccion

$U = (a_1, a_2, a_3 \ldots a_n) \quad V = (b_1, b_2, b_3 \ldots b_n)$

$$U \cdot V = \sum_{i=1}^{j=n} a_i b_i = (a_1 b_1 + a_2 b_2 + a_3 b_3 + a_n b_n)$$

en terminos de sus magnitudes y el angulo relativo de un vector al otro $\quad U \cdot V = \|U\| \, \|V\| \cos\theta$

① Ejemplo $(2, -3, -5, 8) \cdot (-7, -1, 4, 0)$

$(2)(-7) + (-3)(-1) + (-5)(4) + (8)(0)$

$-14 + 3 - 20 + 0 = -31$

② Ejemplo $= (1, -2, 3, -4) \quad (5, -4, 5, 7)$

$(1)(5) + (-2)(-4) + (3)(5) + (-4)(7)$

$5 + 8 + 15 - 28 = 0$

El producto interno $U \cdot V$ se puede interpretar como el producto por la magnitud de V por la magnitud de la proyeccion de U en la direccion de V

Norma o magnitud $\quad \|U\| = \sqrt{U^2 + V^2}$

$\|V\| = (-5, 14, 8, 2) \quad \|V\| = \sqrt{(-5)^2 + (14)^2 + (8)^2 + (2)^2}$

$\|V\| = 17$

$V = (2, 3, 6) \quad \|V\| = \sqrt{(2)^2 + (3)^2 + (6)^2}$

$\|V\| \sqrt{4 + 9 + 36} \quad \|V\| \sqrt{49} \quad \|V\| = 7$

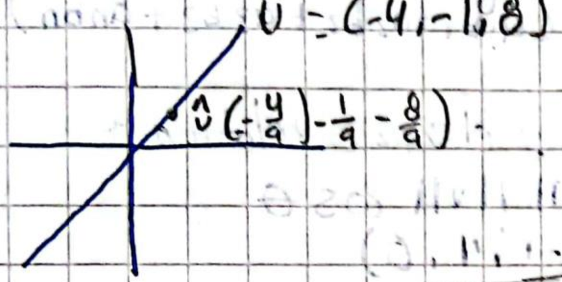
Normalizacion

Es convertir en unitario a un vector que no lo es

$$\hat{u} = \frac{u}{\|u\|} \qquad u = (-4, -1, 8)$$

$$\|u\| = \sqrt{(-4)^2 + (-1)^2 + (8)^2} = 9$$

$$\hat{u}\left[\frac{1}{9}\right](-4, -1, 8) = \hat{u} = \left(\frac{-4}{9}, \frac{-1}{9}, \frac{8}{9}\right)$$



② $V = (2, 3, 6)$

$$\|V\| = \sqrt{(2)^2 + (3)^2 + (6)^2} = 7$$

$$\hat{v}\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$$

Distancia

$$d(u \cdot v) = \|u - v\|$$

$$d(u, v) = \sqrt{[(-1) - (-5)]^2 + [(-11) - (-10)]^2 + [(-9) - (-10)]^2}$$

$$\sqrt{4^2 + (-1)^2 + (1)^2}$$

$$d(u, v) = \sqrt{18}$$

$U = (1, 1, 4) \qquad V = (3, 2, 2)$

$$\sqrt{[(1) - (3)]^2 + [(1) - (2)]^2 + [(4) - (2)]^2}$$

$$-2^2 \quad -1^2 \quad 2^2$$

$$\sqrt{9} = 3$$

Angulo

Coseno del Angulo coplanar entre 2 vectores

$$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$$

Proyeccion

Es el vector que se produce proyectando un vector en la direccion de otro

① $proy(u,v) = \frac{u \cdot v}{\|v\|^2} \cdot v$ ② $proy(v,u) = \frac{v \cdot u}{\|u\|^2} u$

Angulo $\cos\theta = \dfrac{U \cdot V}{\|U\| \, \|V\|}$ $U = (4, -12, 8)$ $V = (2, 1, -2)$

$U \cdot V \; (4)(2) + (-12)(1) + (8)(-2)$
$8 - 12 - 16 = -20$

$\|U\| \sqrt{(4)^2 + (-12)^2 + (8)^2}$ $\|U\| = \sqrt{16 + 144 + 64}$

$\|U\| \sqrt{224} = \sqrt{2^5 \cdot 7}$

$\sqrt{2^5 \cdot 7}$
$\sqrt{2^4 \cdot 2 \cdot 7}$
$\sqrt{2^4} \cdot \sqrt{2 \cdot 7}$
$\sqrt{16} \; \sqrt{14}$
$4\sqrt{14}$

$\|V\| = \sqrt{2^2 + 1^2 + (-2)^2}$
$\|V\| = \sqrt{4 + 1 + 4}$
$\|V\| = \sqrt{9} = 3$

$\cos\theta = \dfrac{-20}{4\sqrt{14} \cdot 3}$

Radicalizar

$\dfrac{-5}{-3\sqrt{14}} \cdot \dfrac{\sqrt{14}}{\sqrt{14}} = \dfrac{-5\sqrt{14}}{3(\sqrt{14})^2}$

$= \dfrac{-5\sqrt{14}}{42} = -0.44543$

$\cos^{-1} = 63.54^{0}$

$U = (6, 2, 3) \qquad V(1, 2, 2)$

$(6)(1) + (2)(2) + (3)(2)$

$6 + 4 + 6$

16

$\|U\| \sqrt{(6)^2 + (2)^2 + (3)^2} \qquad \|V\| \sqrt{(1)^2 + (2)^2 + (2)^2}$

$\|U\| \sqrt{36 + 4 + 9} \qquad \|V\| \sqrt{1 + 4 + 4}$

$\|U\| \sqrt{49} \qquad \|V\| \sqrt{9}$

$\|U\|\, 7 \qquad \|V\| \sqrt{3}$

$\cos \frac{16}{3 \cdot 7} = \frac{16}{21} = 0.7619$

$\cos^{-1} = 63.54$

D M A Scribe

$U_1 = (6, -3, -2) \quad U_2 = (4, 8, 8)$

$U_1 \cdot U_2 = (6)(4) + (-3)(8) + (-2)(8)$
$24 - 24 - 16 = -16$

$\|U_1\| = \sqrt{(6)^2 + (-3)^2 + (-2)^2}$
$\sqrt{36 + 9 + 4}$
$\sqrt{49} = 7$

$\hat{U_1} = \frac{6}{7}, \frac{-3}{7}, \frac{-2}{7}$

$\hat{U_2} = \sqrt{(4)^2 + (8)^2 + (8)^2}$
$\sqrt{16 + 64 + 64}$
$\sqrt{144} = 12$

$\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

$\cos\theta = \frac{-16}{84} = \frac{-4}{21}$

$d(U, V) = \sqrt{[(6)-(4)]^2 + [(-3)-(8)]^2 + [(-2)-(8)]^2}$
$\sqrt{[6-4]^2 + [-3-8]^2 + [-2-8]^2}$
$\sqrt{[2]^2 + [-11]^2 + [-10]^2}$
$\sqrt{4 + 121 + 100}$
$\sqrt{225} = 15$

$V \circ U \quad \frac{-16}{49}(6, -3, -2)$

$\frac{-96}{49}, \frac{48}{49}, \frac{32}{49}$

$U, V \left(\frac{-16}{144}(4, 8, 8) \right)$

$\frac{-64}{144}, -\frac{128}{144}, \frac{-128}{144}$

$-4/9, \frac{-8}{9}, -8/9$

$v_1 = (-9, 6, -2)$ $\quad v_2 = (-8, -4, 1)$

$v_1 \cdot v_2 = (-9)(-8) + (6)(-4) + (-2, 1)$

$72 + (-24) + (-2)$

$72 - 24 - 2 = 46$

$\|v_1\| = \sqrt{(-9)^2 + (6)^2 + (-2)^2}$

$\sqrt{81 + 36 + 4}$

$\sqrt{121} \qquad 11$

$v_1 \to \hat{v} = \frac{v}{\|v\|} \to \frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

$v_2 \to \hat{v} = \sqrt{(-8)^2 + (4)^2 + (1)^2}$

$\sqrt{84 + 16 + 1}$

$\sqrt{81} = 9 \qquad \frac{-8}{9}, \frac{-4}{9}, \frac{1}{9}$

$d(v, v) = \sqrt{[(-9) - (-8)]^2 + [(6) - (-4)]^2 + [(-2) - (1)]^2}$

$\sqrt{[-9 + 8]^2 + [6 + 4]^2 + [-2 - 1]^2}$

$\sqrt{[-1]^2 + [10]^2 + [-3]^2}$

$\sqrt{1 + 100 + 9} \qquad \sqrt{110}$

10.4880

$\cos\theta = \frac{46}{99}$

Proyeccion

$v \cdot v \ \frac{46}{81}(-8, -4, 1) = -368$

$-\frac{368}{81}, \frac{184}{81}, \frac{46}{81}$

$v \cdot v \ \frac{46}{121}(-9, 6, -2)$

$= -\frac{414}{121}, \frac{276}{121}, \frac{92}{121}$

$U_1 = (6, -6, 3)$; $U_2\ (-14, 8, 8)$

$U_1 - U_2\ (6)(-14) + (-6)(8) + (3)(8)$

$-84 - 48 + 24 = \underline{-108}$

$\|U_1\| = \sqrt{(6)^2 + (-6)^2 + (3)^2}$

$\sqrt{36 + 36 + 9}$

$\sqrt{81} = \underline{9}$

$\hat{U_1} = \dfrac{U}{\|U\|} = \dfrac{6}{9}, \dfrac{-6}{9}, \dfrac{3}{9}$

$\hat{U_2}\ \sqrt{(-14)^2 + (8)^2 + (8)^2}$

$\sqrt{196 + 64 + 64}$

$\sqrt{324} = \underline{18}$

$\dfrac{7}{9}, \dfrac{4}{9}, \dfrac{4}{9}$

$d(U.V)\ \sqrt{[(6) - (14)]^2 + [(-6) - (8)]^2 + [(3) - (2)]^2}$

$\sqrt{[6 + 14]^2 + [-6 - 8]^2 + [3 - 8]^2}$

$\sqrt{[20]^2 + [-14]^2 + [-3]^2}$

$\sqrt{400 + 196 + 25}$

$\sqrt{62}$

$\cos\theta = \dfrac{-108}{162}$

$V \cdot V = \dfrac{-108}{81}(6, -6, 3)$ $\quad U \cdot V = \dfrac{108}{81}(-14, 8, 8)$

$\dfrac{648}{81}, \dfrac{648}{81}, \dfrac{324}{81}$ $\quad \dfrac{1512}{324}, \dfrac{-864}{324}, \dfrac{-864}{324}$

$-8, 8, 4$ $\quad 14/3, -8/3, -8/3$

25 Sep 2024

Matriz

Una matriz es un objeto matematico representado por un arreglo rectangular de escalares organizados en filas y columnas de la siguiente forma

$$A\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1m} \\ a_{21}, & a_{22} & a_{23} & \dots & a_{2m} \\ a_{n1}, & a_{n2} & & & a_{nm} \end{bmatrix}$$

Donde

-Los valores a_{ij} son escalares llamados elementos ij

-El conjunto horizontal de escalares se llama Fila

-El conjunto vertical de escalares se llama Columna

-El # total de filas x el # total de columnas se llama tamaño de la matriz $\cdot A_{n \times m}$ $\cdot A \in \mathbb{R}^{n \times m}$

Igualdad de Matrices

Si 2 matrices A y B del mismo tamaño son iguales implica que cada elemento de A es igual a cada elemento de B en ~~cada~~ la misma posicion

Suma Matricial

La operacion de suma entre matrices solo existe si estas son del mismo tamaño

$$A\begin{bmatrix} 2 & -1 \\ -3 & 5 \\ 0 & 8 \end{bmatrix} \qquad B\begin{bmatrix} -5 & 4 \\ -7 & -6 \\ -9 & 2 \end{bmatrix}$$

$$A+B\begin{bmatrix} 2+(-5) & -1+4 \\ -3+(-7 & 5+(-6) \\ 0+9 & 8+2 \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ -10 & -1 \\ 9 & 10 \end{bmatrix}$$

Escalamiento

El escalamiento consiste en un producto escalar K por una matriz

A

$$K\left(-\frac{3}{2}\right) \qquad A\begin{bmatrix} 2 & -5 \\ -3 & 6 \\ 0 & 8 \end{bmatrix} \qquad KA\begin{bmatrix} -\frac{3}{2}(2) & -\frac{3}{6}(-5) \\ -\frac{3}{2}(-3) & -\frac{3}{2}(6) \\ -\frac{3}{2}(0) & -\frac{3}{2}(8) \end{bmatrix}$$

$$\begin{bmatrix} -3 & 15/2 \\ 9/2 & -9 \\ 0 & -12 \end{bmatrix}$$

$$\begin{bmatrix}47\\-32\\-13\end{bmatrix}=\begin{bmatrix}4k_1\\k_1\\-3k_1\end{bmatrix}+\begin{bmatrix}1k_2\\k_2\\-2k_2\end{bmatrix}+\begin{bmatrix}3k_3\\-4k_3\\3k_3\end{bmatrix}$$

$$\begin{matrix}47\\-32\\-13\end{matrix}=\begin{matrix}4k_1 & +1k_2 & +3k_3\\ k_1 & +k_2 & -4k_3\\ -3k_1 & -2k_2 & +3k_3\end{matrix}$$

$k_1 = 6$
$k_2 = -2$
$\underline{k_3 = 9}$

$$\begin{aligned}4k_1 + \cancel{2k_2} + 3k_3 &= 47\\ -2k_1 - \cancel{2k_2} + 8k_3 &= 64\\ \hline 2k_1 + 11k_3 &= 111\end{aligned}$$

$$\begin{aligned}2k_1 + 11k_3 &= 111\\ -2k_1 + 12k_3 &= -120\\ \hline -k_3 &= -9\end{aligned}$$

$\underline{k_3 = 9}$

$$\begin{aligned}4k_1 + 2k_2 + 3k_3 &= 47\\ -3k_1 - 2k_2 + 3k_3 &= -13\\ \hline -k_1 + 6k_3 &= 60\end{aligned}$$

$k_1 + 6(9) = 60$
$k_1 + 54 = 60$
$k_1 = 60 - 54$
$\underline{k_1 = 6}$

$6 + k_2 - 4(9) = -32$
$6 + k_2 - 36 = -32$
$k_2 = -32 + 30$
$k_2 = -2$

$V=(29,9,-58)$ $V_1=(6,0,-1)$ $V_2(1,0,0)$ $V_3(-6,-1,7)$

$$\begin{bmatrix}29\\9\\-58\end{bmatrix}=k_1\begin{bmatrix}6\\0\\-1\end{bmatrix}+k_2\begin{bmatrix}1\\0\\0\end{bmatrix}+k_3\begin{bmatrix}-6\\-1\\7\end{bmatrix}$$

$$\begin{bmatrix}29\\9\\-58\end{bmatrix}=\begin{bmatrix}6k_1\\0\\-k_1\end{bmatrix}+\begin{bmatrix}k_2\\0\\0\end{bmatrix}+\begin{bmatrix}-6k_3\\-k_3\\7k_3\end{bmatrix}$$

$$\begin{bmatrix}29\\9\\-58\end{bmatrix}=\begin{bmatrix}6k_1+k_2-6k_3\\0+0+-k_3\\-k_1+0+7k_3\end{bmatrix}$$

$K_1=-5$
$K_2=5$
$K_3=-9$

$-k_3=9=\underline{-9}$

$-k_1+7(-9)=-58$
$-k_1=-58+63$
$k_1=+5=\underline{-5}$

$6(-5)+k_2-6(-9)=-29$
$-30+k_2+54=29$
$k_2=29-24$
$k_2=\underline{5}$

$V = (-3, -72, -26)$ $v_1(8, 3, 4)$ $v_2(3, -9, -2)$

$$\begin{bmatrix}-3\\-72\\-26\end{bmatrix} = \begin{bmatrix}8k_1\\3k_1\\4k_1\end{bmatrix} + \begin{bmatrix}3k_2\\-9k_2\\-2k_2\end{bmatrix}$$

$K_1 = 3$
$K_2 = 7$

$$\begin{matrix}-3\\-72\\-26\end{matrix} = \begin{matrix}8k_1 + 3k_2\\3k_1 - 9k_2\\4k_1 - 2k_2\end{matrix}$$

$8k_1 + 3k_2 = -3$
$3k_1 - 9k_2 = -72$

$$\begin{aligned}\cancel{8k_1} + 3k_2 &= -3\\ \cancel{-8k_1} + 4k_2 &= 52\\ \hline 7k_2 &= 49\\ k_2 &= 49/7\\ k_2 &= 7\end{aligned}$$

$$\begin{aligned}8k_1 + 3(7) &= -3\\ 8k_1 + 21 &= -3\\ 8k_1 &= -3 - 21\\ &= -24\\ k_1 &= 24/8\\ k_1 &= -3\end{aligned}$$

LINK DE LA PRESENTACION:

https://drive.google.com/file/d/1gZ-SKF2RDnsYAcd7BBXsMyRXQFZClKC0/view?usp=shari